Load Participation in Electricity Markets: Day Ahead and Hour Ahead Market Coupling with Wholesale/Transmission and Retail/Distribution Cost and Congestion Modeling

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***Abstract*—We consider load side participation in electricity markets construed broadly to include energy as well as reserve transactions. Building on our previous work in optimal plug-in-hybrid electric vehicle (PHEV) charging, we use the example of distributed PHEV loads to develop a decision support tool that we believe is generalizable to a broad set of usage types. Whole sale markets operating today in the US clear Day Ahead 24 hour transactions adjusted in subsequent Real Time markets. We recognize that market participation of loads and other resources at the retail/distribution level must be responsive to local distribution network dynamic congestion and marginal line loses. We address the cascading Day Ahead and Hour Ahead markets as well as distribution network congestion and line losses by modeling the interaction of a Load Aggregator (LA) participating in the transmission-level whole sale electricity market with its Smart Microgrid Affiliates (SMAs) connected to the distribution network. We formulate the optimal price and quantity bidding policy of the LA to the Day Ahead market and the bidding of both the LA and SMAs to the Hour Ahead market, conditional upon the energy and reserves scheduled in the clearing of the Day Ahead market. At each Hour Ahead market the LA can sell energy to and buy reserves from its SMAs, using the Hour Ahead markets to settle differences relative to the Day Ahead scheduled quantities. Each of the SMAs can either buy energy from and sell reserves to the LA at a fixed price quoted by the LA and or bid to the Hour Ahead market to bargain on an uncertain, albeit less costly, Hour Ahead market clearing outcome.**

I. Introduction

I.A Effective Load Management and the Integration of Renewable Generation

In the ongoing debate about energy and environmental sustainability, the power system's ability to absorb renewable generation has featured prominently. In this context, the burden of intermittency that accompanies renewable generation has been a major topic of concern [10], [16]. Wind generation variability over time-scales of minutes and inability to dispatch at will over longer time-scales is likely to increase the reserves required to safeguard system stability including regulation service (5 minute time-scale) and operating reserves (15 minute time-scale). Although wind generation is a competitive source of electric energy, depending on the burden that renewable generation places on load following and regulation service reserves, business as usual where such reserves are provided solely by flexible generation resources may not be economically viable. In this case, we will either have to forgo significant renewable generation expansion or rely on efficient load side support.

Several studies claim that a modest increase in regulation service [13] is required to support significant increases in wind generation. However, more recent studies as well as empirical evidence [4]-[6], [8]-[9] indicate that the conclusion of modest regulation service reserve requirements is a significant underestimation. Makarov et al. [8] evaluated a scenario similar to that considered by the CEC, and reported that for a 4,100 MW increment of wind farm nameplate capacity, a maximum increase of 230 MW (5.6%) of regulation-service-down and 500 MW (12.2%) of regulation-service-up would be required! Finally, studies have claimed that with proper geographical diversity in wind farm locations, a sudden loss of wind generation is not a credible event. However, this type of event has occurred in areas with high wind penetration. The Texas balancing authority reported that wind output during certain hours in 2007 was 2,000 MW less then forecasted, and in 2008 wind output unexpectedly dropped 1,300 MW in three hours [3]-[4]. In Europe (e.g., Spain), similar system stability issues due to wind have been experienced [2], [6].

Focusing on alternative sources of fast reserves needed for promoting the clean energy agenda, we argue that efficient load side regulation service support, amongst others by optimal PHEV charging, is achievable by opening up electricity markets to the load side. In this paper we present decision support tools that build upon today's communication capabilities to enable this participation.

I.B Smart Microgrids and Human-in-the-Loop Load Managemen Observing Local Costs and Congestion

Although load management should be able to address individual energy uses and interact with broader utility side of the meter costs and constraints, it is only reasonable to limit the involvement of energy service beneficiaries to express preferences, leaving tedious implementation to an intelligent cyber-enabled automation framework. Such frameworks can be implemented over a large building, a set of buildings or a neighborhood. We envision such cyber physical frameworks to consist of two synergistic layers: (*i*) The *Intelligent Information and Execution* (IIE) layer incorporating a *detailed building/neighborhood model*, a *micro-grid* with sub metering, wireless sensor network and remote actuation capabilities, and an *information and data storage portal* collecting and communicating information internal as well as external to the building (e.g., weather and utility side of the meter costs and requirements). The IIE Layer should also provide a robust execution environment for (*ii*) the *Energy Management Decision Support* (EMDS) layer employing a *suite of stochastic dynamic decision tools* and a *detailed power grid model* of the system on the utility side of the meter. The EMDS layer should provide support for the operation of a Building Side of the Meter Market (BSMM) and its interaction with Utility Side of the Meter Markets (USMMs). The EDMS layer should be able to receive from the IIE layer appropriately processed information on market trends, the building's energy system state and dynamics, allowable sets of actions and occupant preferences. The EMDS layer should be able to compute and return to the IIE desirable cost minimizing actions as well as incentives to elicit occupant authorization of load management degrees of freedom. The IIE should be able to communicate the incentives to occupants and implement EMDS commands with as much fidelity as hardware integrity, building and equipment safety allow.

This paper contributes to such cyber physical framework by developing the market participation decision support capabilities that should reside in the EMDS layer.

I.C Energy and Reserve Market Transactions

We agree with Smith et al. that "operating experience from around the world has shown that a deep, liquid, real-time market is the most economical approach to providing the balancing energy required by variable-output wind plants [16]." In the US, day-ahead, adjustment, and real-time power markets have been operating since the mid 1990s (i.e., PJM, NYISO, NEISO, MISO, SPP, and ERCOT) and PJM has allowed loads to participate in offering capacity reserves since 2006 [7], [11]-[12], [14]. We consider the following market framework where the PHEV load aggregator (LA), or Energy Service Company (ESCo), operates. For a detailed discussion on ESCos please see [1].

For each time period, *t*, market participants make quantity (*Q)* and price (*u*) bids and offers for generation or demand energy, Q_t^E, u_t^E, and for each of the three types of capacity reserves, Q_t^R, $u_t^{RE}, u_t^{RC}; R = 1,2,3$ primary, secondary (regulation service), and tertiary (operating reserves). Primary, secondary and tertiary reserve offers represent stand by capacity that must be deliverable in 3 sec., 5 min., and 15 min., respectively. Primary and secondary reserves respond to central control commands to maintain the market's energy balance in real-time. Moreover, they involve a band of up-and-down capacity (i.e., increment or decrement on command their generation or demand) in the amount of capacity they have offered. As a result, primary and secondary reserve offers are associated with a nominal of generation or consumption rate that is at least equal to the stand by capacity offered. Hence for capacity reserve they offer two prices: an energy and a capacity stand by price.

To elaborate by example, a LA interested in purchasing energy Q_t^E and offering secondary reserves Q_t^R during period *t* will submit to the market two quantities and three prices: $Q_t^E, u_t^E; Q_t^R, u_t^{RE}, u_t^{RC}$. The two prices for the secondary reserves offer correspond to the energy it must consume, and the promise to stand by and respond to an up-or-down command if and when issued by the market operator. More specifically, if the LA offers secondary reserves equal to Q_t^R kilowatts (KW), and the market clearing schedules these reserves, the LA will (i) start the period consuming at the rate of Q_t^R KW and be charged at

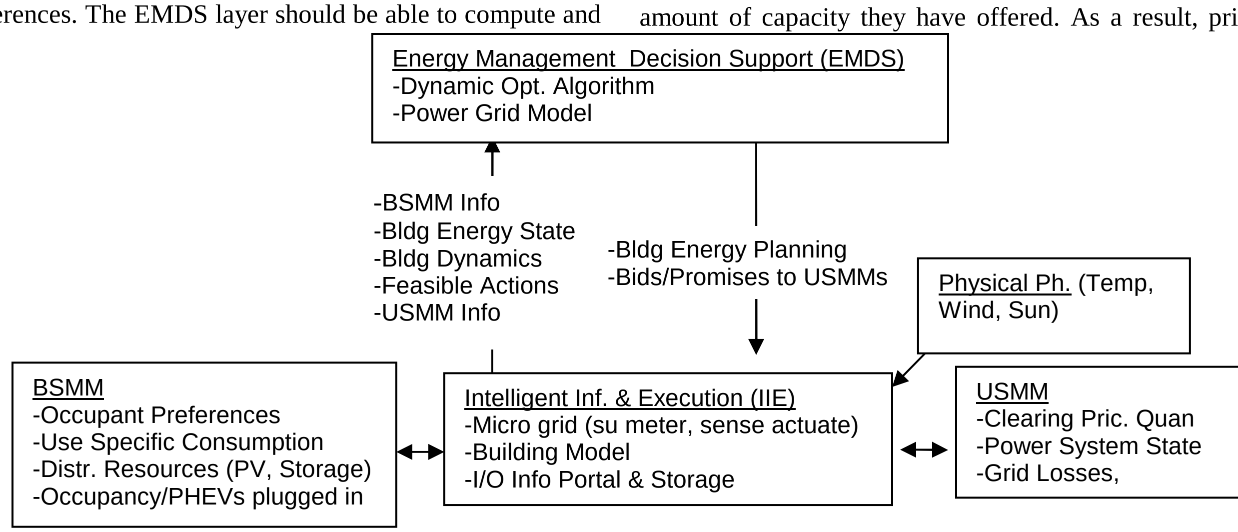

Figure 1. Information Flows: EMDS and IIE Layers, Physical Phenomena, Utility and Building side of Meter Markets (USMM) and (BSMM).

the market energy clearing price of $\tilde{P}_t^E$ per kilowatt-hour (KWh), (ii) be credited at the market secondary reserve clearing price, $\tilde{P}_t^R$ per KWh, and (iii) respond to market operator commands to consume at any level in the interval $\left[0, 2Q_t^R\right]$, moving in the direction that the operator indicates at the rate of $Q_t^R/5$ KW per min.

The market operator receives the LA bids as well as those from all other market participants. The market clears by minimizing costs subject to meeting energy balance and reserve constraints,[1] and determines (*i*) realized market clearing prices denoted by the vector $\tilde{P}_t = \left(\tilde{P}_t^E, \tilde{P}_t^R\right)$ for all participants, and (*ii*) schedules $Q_t^{E,s}$ and $Q_t^{R,s}$ for each participant. Before the market clears, participants do not know the clearing prices or the quantities scheduled. As long as the market is competitive, participants cannot individually affect market clearing prices. However, under symmetric information availability, they all have access to the joint probability distribution (j.p.d.) of clearing prices denoted by $f_t(\tilde{P}_t^E, \tilde{P}_t^R \mid \mathbf{I}_t)$ where $\mathbf{I}_t$ is the state of the system, namely the information available at time t^-, the time the bids and offers for period t are made. Given the above j.p.d., one can evaluate the probability of each of the four key events e_k, k=0,1,2,3 described in equations (1), (2), (3.1), and (3.2) below. The probability depends on the price bid/offer vector $u_t = (u_t^E, u_t^{RE}, u_t^{RC})$ and is denoted by $p_k^t = p_k\left(u_t\right)$; k=0,1,2,3.

Energy bids are accepted as follows:

$$Q_t^{E,s} = \begin{cases} Q_t^E & \text{if } u_t^E \geq \tilde{P}_t^E \text{ i.e., event } e_0 \text{ occurs} \\ 0 & \text{otherwise i.e., event } \overline{e}_0 \text{ occurs} \end{cases} \tag{1}$$

The expected cost of the energy bid Q_t^E is $p_0\left(u_t\right) \underset{\tilde{P}_t|e_0}{E} \tilde{P}_t Q_t^E$.

Reserve offers are accepted and the associated energy is scheduled as follows:

- If event e_1 occurs, the regulation service offer is accepted, i.e.,

$$Q_t^{R,s} = Q_t^R, \text{ if } \left|\tilde{P}_t^E - u_t^{RE}\right| + u_t^{RC} \leq \tilde{P}_t^R \tag{2}$$

- If event e_2 occurs, the secondary reserve offer is rejected, $Q_t^{R,s} = 0$, but the energy component is scheduled, and $Q_t^{E,s} := Q_t^{E,s} + Q_t^R$. Hence, Q_t^R is available for consumption at the expected cost of

$$\underset{\tilde{P}_t|e_2}{E} \tilde{P}_t^E Q_t^R \text{ if} \left|\tilde{P}_t^E - u_t^{RE}\right| + u_t^{RC} > \tilde{P}_t^R \cap u_t^{RE} \leq \tilde{P}_t^E \tag{3.1}$$

- If event e_3 occurs, the secondary reserve offer is rejected, $Q_t^{R,s} = 0$, and the energy component is not scheduled. Hence Q_t^R can not be consumed and there is no associated cost, if

$$\left|\tilde{P}_t^E - u_t^{RE}\right| + u_t^{RC} > \tilde{P}_t^R \cap u_t^{RE} > \tilde{P}_t^E \tag{3.2}$$

Notice that by their definition e_1 and e_2 are disjoint, and hence $\text{Prob}(e_1 \cup e_2) = p_1^t + p_2^t$, a fact used later.

I.D Day-Ahead, Adjustment, and Hour Ahead Markets

There are several related short-term markets that clear in the course of a day. The *day-ahead market* closes to bids at h_1-12-δ on day d-1 and clears at time h_1-12 scheduling simultaneously bids and offers and determining clearing prices for each hour h_j for j=1,2,…,24 of day d. This market performs short-term planning (e.g., hedging, unit commitment, reserve scheduling) functions.

The *adjustment market* that closes at h_1–δ for each hour h_j for j=1,2,…,24 of day d and performs a planning adjustment role. Additional adjustment markets may clear at later times, particularly when significant unexpected events (e.g., power or line outages) occur. We will focus on a single adjustment market for simplicity and without loss of generality.

The *real-time/hour ahead market* that closes at time t and schedules for $t+\Delta_t$ where Δ_t typically equals 5-15 min, although for simplicity we will assume that it is equal to an hour and treat the real time market as an Hour Ahead market. The real time/hour ahead market performs the final adjustments when essentially all uncertainty has realized itself and feasible operational decisions can be made.

Our LA may secure energy purchases and reserve sales in the day-ahead market for each hour of the next day, and it is debited and credited accordingly at h-12. In the Hour Ahead market it can transact incremental energy and reserves and adjust its credit or debit for the fraction of the hour (t, $t+\Delta_t$). The relative timeline is shown in Fig. 2.

h_{-12} h_1-δ h_1 h_2 …h_j t $t+\Delta_t$ h_{j+1}………. h_{24}

Fig. 2. Day-ahead, adjustment, and real-time market timeline.

As a result, the LA makes its decisions, namely its bids and offers in the day-ahead market knowing that it will be able to take corrective action in the Hour Ahead market. Similarly, it makes decisions in the Hour Ahead market, given the outcomes of the day-ahead market and any additional information included in the current state. This coupling of the day-ahead and Hour Ahead markets can be described rigorously as the solution of the following broadly construed stochastic dynamic programs (SDPs), for the Hour Ahead (4), adjustment (5), and day-ahead (6) markets. The formulation below assumes a single participant and does not

[1] We focus here on a single bus disregarding locational price differentiation caused by transmission and other constraints [11], [15].

include the interaction of the LA with its SMAs which we present in the next section.

$$J(\mathbf{I}_t) = \min_{Q_t^E, Q_t^R, u_t^E, u_t^{RE}, u_t^{RC} \in U(I_t)} \{[\underset{\tilde{P}_t|e_0}{E} \tilde{P}_t^E p_0^t Q_t^E + \underset{\tilde{P}_t|e_1 \cup e_2}{E} \tilde{P}_t^E (p_1^t + p_2^t) Q_t^{R,s} - \underset{\tilde{P}_t|e_1}{E} \tilde{P}_t^E p_1^t Q_t^{R,s}]\Delta_t + EJ(\mathbf{I}_{t+1})\} \quad (4)$$

$$J(\mathbf{I}_{h_1-\delta}) = \min_{Q_{h_j}^E, Q_{h_j}^R, u_{h_j}^E, u_{h_j}^{RE}, u_{h_j}^{RC} \in U(I_{h_1-\delta})\forall j} \{\sum_{j=1}^{24}[\underset{\tilde{P}_{h_j}|e_0}{E} \tilde{P}_{h_j}^E p_0^{h_j} Q_{h_j}^E + \underset{\tilde{P}_{h_j}|e_1 \cup e_2}{E} \tilde{P}_{h_j}^E (p_1^{h_j} + p_2^{h_j}) Q_{h_j}^R - \underset{\tilde{P}_{h_j}|e_1}{E} \tilde{P}_{h_j}^E p_1^{h_j} Q_{h_j}^R] + EJ(\mathbf{I}_1)\} \quad (5)$$

$$J(\mathbf{I}_{h_1-12-\delta}) = \min_{Q_{h_j}^E, Q_{h_j}^R, u_{h_j}^E, u_{h_j}^{RE}, u_{h_j}^{RC} \in U(I_{h_1-12-\delta})\forall j} \{\sum_{j=1}^{24}[\underset{\tilde{P}_{h_j}|e_0}{E} \tilde{P}_{h_j}^E p_0^{h_j} Q_{h_j}^E + \underset{\tilde{P}_{h_j}|e_1 \cup e_2}{E} \tilde{P}_{h_j}^E (p_1^{h_j} + p_2^{h_j}) Q_{h_j}^R - \underset{\tilde{P}_{h_j}|e_1}{E} \tilde{P}_{h_j}^E p_1^{h_j} Q_{h_j}^R] + EJ(\mathbf{I}_{h_1-\delta})\} \quad (6)$$

$U(I_t)$ is the allowable decision set given the state or information of the system at time period t, and the evolution of information is described below.

$\mathbf{I}_{h_1-12-\delta}$ is the relevant information or state vector just before day-ahead market closes. It contains the jpds of hourly clearing prices, PHEV charging demand, local line capacities, as well as other power system information such as outages, wind farm output forecasts.

$\mathbf{I}_{h_1-\delta}$ is the relevant information or state vector just before the adjustment market closes. It contains the results of the clearing of the day-ahead market, including hourly clearing prices and scheduled energy consumption and reserve offerings. In addition, it includes jpds and system information as described above and updated at h_1-δ.

$\mathbf{I}_1$ is the relevant information or state vector just before the first Hour Ahead market closes. It contains (i) the results of the clearing of the day ahead and adjustment markets, including hourly clearing prices and scheduled energy consumption and reserve offerings, (ii) the jpds of future Hour Ahead market clearing prices, PHEV charging demand, local line capacities, (iii) other power system information, and (iv) the actual local line capacities during period t = 0, and the actual uncharged battery capacity and desired departure times of PHEVs plugged-in at time t = 0.

$\mathbf{I}_2$ is the relevant information or state vector just before the second Hour Ahead market closes. It contains all the relevant information in $\mathbf{I}_1$ updated by the clearing of the previous Hour Ahead market and the actual local line capacities during period t = 1 and PHEVs plugged-in at time t = 1. We generalize by defining a function V_t presented in (7).

$$\mathbf{I}_{t+1} = V_t(\mathbf{I}_t, \text{new info revealed during period } t) \quad (7)$$

We continue below by describing the Hour Ahead market problem for both the LA and each of the Smart Microgrid Affiliates (SMAs) that are all subject to the same energy and reserve clearing prices. Each SMA, however, (*i*) must abide by specific local congestion constraints that are associated with a specific transformer such as in the case of a single feeder line of the distribution network, and (*ii*) is subject to location specific marginal line losses.

II. DAY AND HOUR AHEAD MARKET COUPLING: WHOLE SALE AND RETAIL TRANSACTIONS

For simplicity of exposition, but without loss of generality, we disregard the adjustment market, considering instead the day ahead market and the hour ahead market representing the essence of the real time market. This renders $t \in \{1, 2, ..., 24\}$ and Δ_t = 1 hour.

II.A *Masterproblem: LA Secures Energy and Reserves in the Day Ahead and Provides to SMAs in the Hour Ahead*

The LA bids to the day ahead market and secures from the clearing of the market hourly energy purchases and reserve sales $Q_t^{E,s}, Q_t^{R,s} \quad \forall t \in \{1, 2, ..., 24\}$ scheduled at the clearing of the day ahead market where the LA bids optimally by solving the following DP involving decisions at times:

h_1-12-δ=1-12-δ

$t \in \{1, 2, ..., 24\}$

Note that the decisions at the day ahead market differ from the decisions at the hour ahead market:

- At the day ahead market the LA bids hourly prices and quantities for hourly energy and reserve quantities.
- At the hour ahead market, the LA (*i*) sells energy and buys reserves from its SMAs according to prices $\hat{P}_t^E, \hat{P}_t^R$ that it selects, and (*ii*) sells/buys to the hour ahead market the surplus or deficit relative to the day ahead secured quantities $Q_t^{E,s}, Q_t^{R,s}$

More specifically, the LA solves the following stochastic DP problem at the day ahead market to obtain optimal bidding policies:

$$J(\mathbf{I}_{1-12-\delta}) = \min_{Q_h^E, Q_h^R, u_h^E, u_h^{RE}, u_h^{RC} \in U(I_{1-12-\delta})\forall h} \{\sum_{h=1}^{24}[\underset{\tilde{P}_h|e_0}{E} \tilde{P}_h^E p_0^h Q_h^E + \underset{\tilde{P}_h|e_1 \cup e_2}{E} \tilde{P}_h^E (p_1^h + p_2^h) Q_{h_j}^R - \underset{\tilde{P}_h|e_1}{E} \tilde{P}_h^E p_1^h Q_h^R] + EJ(\mathbf{I}_1)\}$$

However, the day ahead problem requires knowledge of the cost to go in the hour ahead markets $J(\mathbf{I}_1)$. Defining by $\hat{\mathbf{Q}}_t^E = \sum_{i,\tau} \hat{Q}_{i,t}^E(\tau), \hat{\mathbf{Q}}_t^R = \sum_{i,\tau} \hat{Q}_{i,t}^R(\tau)$ the energy sales to and reserve purchases from its SMAs the LA solves the hour ahead problem to determine optimal $\hat{P}_t^E, \hat{P}_t^R$ prices. Note that in the hour ahead market the PA is a *price taker* . It sells back to the hour ahead market excess energy relative to the quantities secured in the day ahead but not demanded by the SMAs during the hour ahead market and buys back from the hour ahead market excess reserves scheduled when the day ahead market cleared but not supplied by the SMAs during the hour ahead market. Therefore the LA evaluates the cost to go by solving the backward hour ahead DP algorithm:

$$J(\mathbf{I}_t) = \min_{\hat{P}_t^E, \hat{P}_t^R} E\{\tilde{P}_t^E[\hat{\mathbf{Q}}_t^E - Q_t^{E,s}] - \hat{P}_t^E \hat{\mathbf{Q}}_t^E$$
$$-\tilde{P}_t^R[\hat{\mathbf{Q}}_t^R - Q_t^{R,s}] + \hat{P}_t^R \hat{\mathbf{Q}}_t^R + J(\mathbf{I}_{t+1})\}$$

where $\hat{Q}_{i,t}^E(\tau), \hat{Q}_{i,t}^R(\tau)$ are determined by the SMAs in venues/neighborhoods i=1,2,…M by solving the following related sub-problems in response to the prices $\hat{P}_t^E, \hat{P}_t^R$ decided by the LA.

Note that the LA incurs a cost in the day ahead market when it buys energy, but realizes an income at the hour ahead market when it sells that energy to the SMAs.

II.B *SMA Subproblems: Hour Ahead Decision solved by each SMA*

Indices and Problem Parameters

t : Decision period.

i : i^{th} Smart Grid Affiliate,

τ : Index of plugged-in PHEV departure classes.

N : Number of time periods in the finite horizon. N=24 in this approximation of real time market by hour ahead market

c: Penalty ($ per KWh) of uncharged energy at time of PHEV departure.

r : Charging rate (KW) of each PHEV.

λ_N^τ : Marginal costs ($ per KWh) of charging PHEVs with departure class outside of the horizon (i.e., $\tau > N$).

State and Decision Variabes

$n_{i,t}^\tau, x_{i,t}^\tau$: Number i^{th} SMA PHEVs and their uncharged energy (KWh) plugged-in at the beginning of period *t*, in departure class *τ*.

$\hat{Q}_{i,t}^E(\tau), \hat{Q}_{i,t}^R(\tau)$: i^{th} SMA Energy rate purchased from (KW) and regulation service capacity sold to (KW), respectively, the PA during period *t*. Intended for charging PHEVs with uncharged energy $x_{i,t}^\tau$.

$Q_{i,t}^E(\tau), Q_{i,t}^R(\tau)$: i^{th} SMA Energy rate requested (KW) and regulation service capacity offered (KW), respectively, to the hour ahead market during period *t*. Intended for charging PHEVs with uncharged energy $x_{i,t}^\tau$.

$u_{i,t}^E(\tau)$: i^{th} SMA Energy bid price for $Q_{i,t}^E(\tau)$.

$(u_{i,t}^{RE}(\tau), u_{i,t}^{RC}(\tau))$: i^{th} SMA Energy and capacity price offered, respectively, for $Q_{i,t}^R(\tau)$.

$\mathbf{I}_{i,t}$: i^{th} SMA Relevant information or state vector at time *t*. This includes j.p.d.s of future PHEV demand, line capacities, and Hour Ahead market clearing price p.d.f.s conditional upon physical phenomena such as weather forecasts and the overall power system state including known plant outages and wind output forecasts that may affect reserve requirements, bids by other market participants and ultimately clearing prices. In addition, it contains SMA location-specific distribution capacity available for PHEV battery charging $\left(\hat{C}_{i,t}^{\max}\right)$, the factor of marginal line losses $\left(m_{i,t} = 1/(1+\text{Marg.Losses}_{i,t})\right)$ that converts energy and reserves at the exit of the whole sale market transmission system to the quantity after losses at the SMA venue. Finally, it includes quantities scheduled and clearing prices observed in all hourly markets that closed previously, LA prices $\hat{P}_t^E, \hat{P}_t^R$, and most importantly the number of PHEVs plugged-in SMA *i* and their uncharged capacity $n_{i,t}^\tau, x_{i,t}^\tau$.

Random Variables and Density Functions: Load Aggregator and Smart Microgrid Affiliates (SMAs)

E_t: Expectation operator conditional on information available at the beginning of period *t*.

$\tilde{P}_t^E, \tilde{P}_t^R$: Random variables for the Hour Ahead market clearing prices for energy and regulation service during period *t*.

$\tilde{\Delta}n_{i,t}^\tau, \tilde{\Delta}x_{i,t}^\tau$: Random variables indicating the number of PHEVs and their uncharged energy (KWh) expected to plug-in at i^{th} SMA during period *t* in departure class *τ*.

$p_{i,k}^{t,\tau}$: The probabilities of the four key events *k*=0,1,2,3 defined in detail in Section I.C in relation to price bid vector $u_{i,t}^\tau = (u_{i,t}^{E,\tau}, u_{i,t}^{RE,\tau}, u_{i,t}^{RC,\tau})$

$\tilde{1}_{i,RS}^{t,\tau}$: A random indicator function dependent upon price bid vector $u_{i,t}^\tau$ that equals 1 with probability $p_{i,\alpha}^{t,\tau} = p_{i,1}^{t,\tau} + p_{i,2}^{t,\tau}$.

System Dynamics: Smart Microgrid Affiliates (SMAs)

Up-and-down reserves, including regulation service, are exercised by the market operator so that over a half hour or longer period energy neutrality is maintained. As a result, we can write the system dynamics (8)-(11).

$$n_{i,t+1}^\tau = n_{i,t}^\tau + \tilde{\Delta}n_{i,t}^\tau \quad (8)$$

$$x_{i,t+1}^\tau = x_{i,t}^\tau + \tilde{\Delta}x_{i,t}^\tau - m_{i,t}[Q_{i,t}^E(\tau) + \hat{Q}_{i,t}^E(\tau) + \hat{Q}_{i,t}^R(\tau) + \tilde{1}_{i,RS}^{t,\tau} Q_{i,t}^R(\tau)] \quad (9)$$

$$n_{i,t}^\tau = x_{i,t}^\tau = 0 \quad (10)$$

$$\mathbf{I}_{i,t+1} = V_{i,t}(\mathbf{I}_{i,t}, \tilde{P}_t^E, \tilde{P}_t^R, n_{i,t+1}^\tau, x_{i,t+1}^\tau, m_{i,t}, \hat{C}_{i,t+1}^{\max}, \text{new info}) \quad (11)$$

Allowable Decisions: Smart Microgrid Affiliates (SMAs)

The LA must follow market rules to make sure that its energy bid and regulation service offer are realizable. This requires that two constraints (12)-(13) on the maximal consumption rate (i.e., the requested energy rate plus *twice* the offered regulation service). *First*, the excess SMA location specific capacity should be sufficient to support the maximal consumption rate. *Second*, there must be enough plugged-in PHEVs to absorb the maximal charging rate. Note that (12) couples the departure classes. In addition, the allowable control set includes non-negativity constraints on all the state and decision variables.

$$\Sigma_{\tau} m_{i,t}[Q^{E}_{i,t}(\tau)+\hat{Q}^{E}_{i,t}(\tau)+2(Q^{R}_{i,t}(\tau)+\hat{Q}^{R}_{i,t}(\tau))]\leq \hat{C}^{\max}_{i,t} \qquad (12)$$

$$m_{i,t}[Q^{E}_{i,t}(\tau)+\hat{Q}^{E}_{i,t}(\tau)+2(Q^{R}_{i,t}(\tau)+\hat{Q}^{R}_{i,t}(\tau))]\leq rn^{\tau}_{i,t} \quad \tau>t \qquad (13)$$

$$m_{i,t}[Q^{E}_{i,t}(\tau)+\hat{Q}^{E}_{i,t}(\tau)+Q^{R}_{i,t}(\tau)+\hat{Q}^{R}_{i,t}(\tau)]\leq x^{\tau}_{i,t} \quad \tau>t \qquad (14)$$

Bellman Equation: Smart Microgrid Affiliates(SMAs)

Decisions are made at the beginning of each time period t employing the information or state, $\mathbf{I}_{i,t}$, including probability distributions and past clearing prices. Letting $\mathbf{u}_{i,t}$ be the vector of all the decision variables that have to be decided at time t, the Bellman Equation can be written as (15).

$$J\left(\mathbf{I}_{i,t}\right)=\min_{\mathbf{u}_{i,t}\in U_{i,t}(\mathbf{I}_{i,t})} E_t\begin{bmatrix} g_{i,t}\left(\mathbf{I}_{i,t},\mathbf{u}_{i,t},\tilde{P}^{E}_{t},\tilde{P}^{R}_{t},t\right)\\ +J_{i,t+1}\left(\mathbf{I}_{i,t+1}\right)\end{bmatrix} \qquad (15)$$

with boundary condition $J\left(\mathbf{I}_{i,N}\right)=cx^{N}_{i,N}+\sum_{\tau>N}x^{\tau}_{i,N}\lambda^{\tau}_{i,N}$

Where $Eg_{i,t}\left(\mathbf{I}_{i,t},\mathbf{u}_{i,t},\tilde{P}^{E}_{t},\tilde{P}^{R}_{t},t\right)=$

$$\Sigma_{\tau}\{\underset{\tilde{P}_t|e_0}{E}\tilde{P}^{E}_{i,t}p^{t,\tau}_{i,0}Q^{E}_{i,t}(\tau)+\hat{P}^{E}_{i,t}\hat{Q}^{E}_{i,t}(\tau)+\underset{\tilde{P}_t|e_1\cup e_2}{E}\tilde{P}^{E}_{t}p^{t,\tau}_{i,\alpha}Q^{R}_{i,t}(\tau)$$

$$-\underset{\tilde{P}_t|e_1}{E}\tilde{P}^{R}_{t}p^{t,\tau}_{i,1}Q^{R}_{i,t}(\tau)+\left(\hat{P}^{E}_{t}-\hat{P}^{R}_{t}\right)\hat{Q}^{R}_{i,t}(\tau)\}+cx^{t}_{i,t}$$

III Solution Approach

Solution of the Cascading markets problem defined above requires the simultaneous solution of several linked stochastic dynamic problems. The assumption that the SMAs do not participate in the day ahead market is a reasonable assumption that is not motivated merely by the desire to simplify the problem. It make a lot of sense for the Load Aggregator (LA) to undertake the hedging function through its participation in the day ahead market and then distribute the scheduled quantities to its SMAs through the hour ahead markets when the SMAs know the value of the local constrain, $\hat{C}^{\max}_{i,t}$ and the marginal losses factor $m_{i,t}$.

We have already addressed successfully the solution of the specific SMA hour ahead problem using a hybrid Optimal Open Loop approximation employing Multiple Stochastic Programming for finite look ahead [1], [5]. Using the aforementioned solution building block to obtain efficient solutions to the multiple SMA sub problems we can employ the following algorithm:

- start with a guess at the LA hour ahead solution for energy and reserve prices, $\hat{P}^{E}_{t},\hat{P}^{R}_{t}$ possibly setting them equal to the expected value of the hour ahead clearing prices.
- Solve the SMA hour ahead problems using the approximation in [1],[5] to obtain tentative $\hat{Q}^{E}_{i,t}(\tau),\hat{Q}^{R}_{i,t}(\tau)$ values.
- Explore a direction of improvement in the guess of prices $\hat{P}^{E}_{t},\hat{P}^{R}_{t}$ in the LA hour ahead problem by comparing the values of $\hat{\mathbf{Q}}^{E}_{t}=\sum_{i,\tau}\hat{Q}^{E}_{i,t}(\tau),\hat{\mathbf{Q}}^{R}_{t}=\sum_{i,\tau}\hat{Q}^{R}_{i,t}(\tau)$ from the SMA sub problem solutions to the scheduled $Q^{E,s}_{t},Q^{R,s}_{t}$ values. Increase or decrease the prices respectively if the SMA sub problems requested collectively more energy and sold more reserves than the LA had scheduled in the clearing of the day ahead problem, and vice versa. This will also indicate whether higher or lower quantities could have been scheduled to advantage in the day ahead market.
- Repeat this Lagrangian-relaxation-like sub gradient method to iteratively improve the day ahead and hour ahead solutions of the complex stochastic dynamic programming problem.

II. Conclusions and Future Work

We formulated the cascading day ahead and hour ahead markets problem incorporating local distribution constraints and costs into a near optimal decision support algorithm for efficient participation of load into existing wholesale and anticipated retail power markets that the advent of the smart grid may enable. To fix ideas and to address an important new load category we focused on a specific class of load associated with PHEV battery charging. However, similar, and certainly not more complex models can be developed and utilized to address a broad set of electricity uses such as lighting, electric appliances and HVAC. In future work we intend to address more general load participation in power markets and develop more efficient solution techniques relying on robust optimization methodologies and discrete event simulation.

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